Solutions to Exercise 1A

(Joseph Yeo, Teh Keng Seng, Loh Cheng Yee and Ivy Chew's New Syllabus Additional Mathematics, 9th Edition – ISBN 9789812374998)

Solved by:
Dr Lee Chu Keong
(ascklee@gmail.com)

mathematics-dl.org

Exercise 1A (p. 6)

Question 1(a)

$$y = 5 - 2x \ldots (1)$$
$$x^2 + y^2 = 5 \ldots (2)$$

Subsituting (1) into (2):

$$x^2 + (5 - 2x)^2 = 5$$
$$x^2 + 25 - 20x + 4x^2 = 5$$
$$5x^2 - 20x + 20 = 0$$

Divide throughout by 5:

$$x^2 - 4x + 4 = 0$$
$$(x - 2)^2 = 0$$
$$x = 2 \ldots (3)$$

Substituting (3) into (1):

$$y = 5 - 2(2)$$
$$= 5 - 4$$
$$= 1$$

Question 1(b)

$$y = 2 - x \ldots (1)$$
$$x(x + y) = 5 - 3y^2 \ldots (2)$$

Substituting (1) into (2):

$$x(x + 2 - x) = 5 - 3(2 - x)^2$$
$$x(2) = 5 - 3(4 + x^2 - 4x)$$
$$2x = 5 - (12 + 3x^2 - 12x)$$
$$2x = 5 - 12 - 3x^2 + 12x$$
$$3x^2 - 10x + 7 = 0$$
$$(3x - 7)(x - 1) = 0$$

$x = \frac{7}{3}$	$x = 1$
Substituting into equation (1):	
$y = 2 - \frac{7}{3}$	$y = 2 - 1$ $= 1$

mathematics-dl.org

$= -\frac{1}{3}$	

Question 1(c)

$$2x = 1 - 3y$$

$$\therefore \quad x = \frac{1}{2} - \frac{3}{2}y \ldots (1)$$

$$3y^2 - x^2 = 2 \ldots (2)$$

Substituting (1) into (2):

$$3y^2 - \left(\frac{1}{2} - \frac{3}{2}y\right)^2 = 2$$

$$3y^2 - \left(\frac{1}{4} + \frac{9}{4}y^2 - \frac{3}{2}y\right) = 2$$

$$\frac{3}{4}y^2 + \frac{3}{2}y - \frac{9}{4} = 0$$

Multiply throughout by 4:

$$3y^2 + 6y - 9 = 0$$

Divide throughout by 3:

$$y^2 + 2y - 3 = 0$$

$$(y + 3)(y - 1) = 0$$

$y = -3$	$y = 1$
Substituting into equation (1):	
$x = \frac{1}{2} - \frac{3}{2}(-3)$ $= \frac{1}{2} + \frac{9}{2}$ $= 5$	$x = \frac{1}{2} - \frac{3}{2}(1)$ $= \frac{1}{2} - \frac{3}{2}$ $= -1$

Question 1(d)

$$y = 14 - 3x \ldots (1)$$

$$x^2 + y^2 = 34 \ldots (2)$$

Substituting (1) into (2):

$$x^2 + (14 - 3x)^2 = 34$$

$$x^2 + (196 + 9x^2 - 84x) = 34$$

$$10x^2 - 84x + 162 = 0$$

mathematics-dl.org

Divide throughout by 2:

$$5x^2 - 42x + 81 = 0$$

$$(5x - 27)(x - 3) = 0$$

$x = \frac{27}{5}$	$x = 3$
Substituting into equation (1):	
$y = 14 - 3\left(\frac{27}{5}\right)$ $= 14 - 16\frac{1}{5}$ $= -2\frac{1}{5}$	$y = 14 - 3(3)$ $= 14 - 9$ $= 5$

Question 1(e)

$$x = y + 3 \ldots (1)$$

$$3y^2 = x^2 + 2xy + 1 \ldots (2)$$

Substituting (1) into (2):

$$3y^2 = (y + 3)^2 + 2(y + 3)y + 1$$

$$3y^2 = (y^2 + 9 + 6y) + (2y^2 + 6y) + 1$$

$$3y^2 = 3y^2 + 12y + 10$$

$$12y = -10$$

$$y = -\frac{10}{12}$$

$$= -\frac{5}{6} \ldots (3)$$

Substituting (3) into (1):

$$x = -\frac{5}{6} + 3$$

$$= -\frac{5}{6} + \frac{18}{6}$$

$$= \frac{13}{6}$$

$$= \frac{12}{6} + \frac{1}{6}$$

$$= 2\frac{1}{6}$$

mathematics-dl.org

Question 1(f)

$$y = 7 - 4x \ldots (1)$$

$$4x^2 - 4xy + y^2 = 1 \ldots (2)$$

Substituting (1) into (2):

$$4x^2 - 4x(7 - 4x) + (7 - 4x)^2 = 1$$

$$4x^2 - (28x - 16x^2) + (49 + 16x^2 - 56x) = 1$$

$$36x^2 - 84x + 48 = 0$$

Divide throughout by 12:

$$3x^2 - 7x + 4 = 0$$

$$(3x - 4)(x - 1) = 0$$

$x = \frac{4}{3}$	$x = 1$
Substituting into equation (1):	
$y = 7 - 4\left(\frac{4}{3}\right)$ $= 7 - \frac{16}{3}$ $= 2\frac{1}{3}$	$y = 7 - 4(1)$ $= 7 - 4$ $= 3$

Question 1(g)

$$2x = 13 - 3y \ldots (1)$$

$$(2x)y + 5y^2 - (2x)^2 = 41 \ldots (2)$$

Substituting (1) into (2):

$$(13 - 3y)y + 5y^2 - (13 - 3y)^2 = 41$$

$$13y - 3y^2 + 5y^2 - (169 + 9y^2 - 78y) = 41$$

$$13y + 2y^2 - 169 - 9y^2 + 78y = 41$$

$$-7y^2 + 91y - 210 = 0$$

Divide throughout by -7:

$$y^2 - 13y + 30 = 0$$

$$(y - 3)(y - 10) = 0$$

$y = 3$	$y = 10$
Substituting into equation (1):	
$2x = 13 - 3(3)$	$2x = 13 - 3(10)$

mathematics-dl.org

$= 13 - 9$ $= 4$ $x = \frac{4}{2}$ $= 2$	$= 13 - 30$ $= -17$ $x = -\frac{17}{2}$ $= -8\frac{1}{2}$

Question 1(h)

$$x = 7y + 2 \ldots (1)$$
$$(7y + 2)^2 = 34y^2 + 7(7y + 2)y - 16$$
$$49y^2 + 4 + 28y = 34y^2 + 49y^2 + 14y - 16$$
$$0 = 34y^2 - 14y - 20$$

Divide throughout by 2:

$$0 = 17y^2 - 7y - 10$$
$$= (17y + 10)(y - 1)$$

$y = -\frac{10}{17}$	$y = 1$
Substituting into equation (1):	
$x = 7\left(-\frac{10}{17}\right) + 2$ $= -\frac{70}{17} + 2$ $= -\frac{70}{17} + \frac{34}{17}$ $= -\frac{36}{17}$ $= -\frac{34}{17} - \frac{2}{17}$ $= -2\frac{2}{17}$	$x = 7(1) + 2$ $= 7 + 2$ $= 9$

Question 2 (a)

$$2y = 3x - 1 \ldots (1)$$
$$\frac{8x}{2y} + \frac{18y}{2x} = 15$$

mathematics-dl.org

$$\frac{8x}{2y}+\frac{9(2y)}{2x}=15 \ldots (2)$$

Substituting (1) into (2):

$$\frac{8x}{3x-1}+\frac{9(3x-1)}{2x}=15$$

$$\frac{8x(2x)+9(3x-1)^2}{2x(3x-1)}=15$$

$$\frac{16x^2+9(9x^2+1-6x)}{2x(3x-1)}=15$$

$$\frac{16x^2+81x^2+9-54x}{6x^2-2x}=15$$

Cross multiplying:

$$16x^2+81x^2+9-54x=90x^2-30x$$

$$7x^2-24x+9=0$$

$$(7x-3)(x-3)=0$$

$$x=\frac{3}{7} \quad or \quad x=3$$

$$2y=3\left(\frac{3}{7}\right)-1 \quad or \quad 2y=3(2)-1$$

$$=\frac{9}{7}-1 \quad or \quad =7-1$$

$$=\frac{2}{7} \quad or \quad =2$$

$$y=\frac{1}{7} \quad or \quad y=1$$

Question 2(b)

$$\frac{4}{2x}+\frac{3}{y}=13 \quad \ldots (1)$$

$$2x=2-3y \quad \ldots (2)$$

Substituting (2) into (1):

$$\frac{4}{2-3y}+\frac{3}{y}=13$$

$$\frac{4y+3(2-3y)}{y(2-3y)}=13$$

$$4y+6-9y=13(2y-3y^2)$$

$$6-5y=26y-39y^2$$

mathematics-dl.org

$$39y^2 - 31y + 6 = 0$$
$$(13y - 6)(3y - 1) = 0$$

$y = \frac{6}{13}$	$y = 1$
Substituting into equation (2):	
$2x = 2 - 3\left(\frac{6}{13}\right)$ $= 2 - \frac{18}{13}$ $= \frac{8}{13}$ $x = \frac{4}{13}$	$2x = 2 - 3(1)$ $= 2 - 3$ $= -1$ $x = -\frac{1}{2}$

Question 3

Substituting $x = 2$ into the equation:

$$4a + 2b + 1 = 1 \quad \ldots(1)$$

Substituting $x = 3$ into the equation:

$$9a + 3b + 1 = 4 \quad \ldots(2)$$

From (1):

$$4a + 2b = 0$$
$$2b = -4a$$
$$\therefore \ b = -2a \quad \ldots(3)$$

Substituting (3) into (2):

$$9a + 2(-2a) + 1 = 4$$
$$9a - 6a = 4 - 1$$
$$3a = 3$$
$$\therefore \ a = 1$$
$$b = -2a$$
$$= -2(1)$$
$$= -2$$

Question 4

$$2\pi r + 2\pi R = 38\pi$$
$$2\pi(r + R) = 38\pi$$

mathematics-dl.org

Dividing throughout by 2π:

$$r + R = 19$$
$$r = 19 - R \ldots (1)$$

Dividing throughout by π:

$$\pi r^2 + \pi R^2 = 193\,\pi$$
$$r^2 + R^2 = 193 \ldots (2)$$

Substituting (1) into (2):

$$(19 - R)^2 + R^2 = 193$$
$$361 + R^2 - 38R + R^2 = 193$$
$$2R^2 - 38R + 168 = 0$$
$$R^2 - 19R + 84 = 0$$
$$(R - 12)(R - 7) = 0$$

$R = 12$
$r = 19 - 12$
$= 7$

$R = 7$
$r = 19 - 7$
$= 12$

Question 5

$$x + y = 3 \ldots (1)$$
$$xy = 1.25 \ldots (2)$$

From (1):

$$y = 3 - x \ldots (3)$$

Substituting (3) into (2):

$$x(3 - x) = 1.25$$
$$3x - x^2 = 1.25$$
$$0 = x^2 - 3x + 1.25$$
$$= 4x^2 - 12x + 5$$
$$(2x - 5)(2x - 1) = 0$$
$$x = \frac{5}{2} \quad or \quad x = \frac{1}{2}$$
$$y = \frac{1}{2} \quad or \quad y = \frac{5}{2}$$

Question 6

$$a + 2b = 2$$

mathematics-dl.org

$$\therefore \quad a = 2 - 2b \quad \ldots (1)$$

$$b + 2a^2 = 10 \quad \ldots (2)$$

Substituting (2) into (1):

$$b + 2(2 - 2b)^2 = 10$$

$$b + 2(4 + 4b^2 - 8b) = 10$$

$$b + 8 + 8b^2 - 16b = 20$$

$$8b^2 - 15b - 2 = 0$$

$$(8b + 1)(b - 2) = 0$$

$b = -\frac{1}{8}$	$b = 2$
Substituting into equation (1):	
$a = 2 - 2\left(-\frac{1}{8}\right)$ $= 2 + \frac{1}{4}$ $= 2\frac{1}{4}$	$a = 2 - 2(2)$ $= 2 - 4$ $= -2$

$$b = -\frac{1}{8} \quad or \quad b = 2$$

Question 7

$$y - x = 1\frac{1}{2}$$

$$y = \frac{3}{2} + x \ldots (1)$$

$$x^2 + y^2 = 9\frac{1}{8} \ldots (2)$$

Substituting (1) into (2):

$$x^2 + \left(\frac{3}{2} + x\right)^2 = 9\frac{1}{8}$$

$$x^2 + \left(\frac{9}{4} + x^2 + 3x\right) = 9\frac{1}{8}$$

$$2x^2 + 3x - \frac{55}{8} = 0$$

Multiplying throughout by 8:

$$16x^2 + 24x - 55 = 0$$

mathematics-dl.org

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-24 \pm \sqrt{24^2 - 4(16)(-55)}}{2(16)}$$
$$= \frac{-24 \pm \sqrt{576 + 3520}}{32}$$
$$= \frac{-24 \pm \sqrt{4096}}{32}$$
$$= \frac{-24 \pm 64}{32}$$

$x = 1.25\ (accept) \quad or \quad -2.75\ (rejected - \text{question calls for two} + ve \text{ numbers})$

$y = 2.75\ (accept) \quad or \quad -1.25\ (rejected - \text{question calls for two} + ve \text{ numbers})$

Check difference:

$$2.75 - 1.25 = 1.5 \quad ✓$$

Check sum-of-squares:

$$2.75^2 + 1.25^2 = 7.5625 + 1.5625$$
$$= 9.125 \quad ✓$$

Question 8

$$2x^2 + 3y^2 = 110 \ldots (1)$$
$$2y + 3x = 8 \ldots (2)$$

From (2):

$$2y = 8 - 3x$$
$$y = 4 - \frac{3}{2}x \ldots (3)$$

Substituting (3) into (1):

$$2x^2 + 3\left(4 - \frac{3}{2}x\right)^2 = 110$$
$$2x^2 + 3\left(16 + \frac{9}{4}x^2 - 12x\right) = 110$$
$$\frac{8}{4}x^2 + \left(48 + \frac{27}{4}x^2 - 36x\right) = 110$$

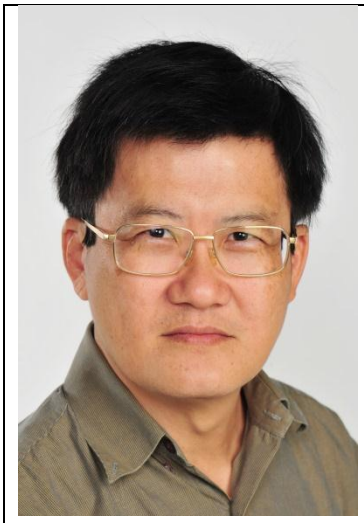
mathematics-dl.org

$$\left(\frac{8}{4}x^2 + \frac{27}{4}x^2\right) - 36x + (48 - 110) = 0$$

$$\frac{35}{4}x^2 - 36x - 62 = 0$$

Multiplying throughout by 4:

$$35x^2 - 144x - 248 = 0$$



Dr. Lee is an experienced teacher who has taught at the Singapore and Temasek Polytechnics. He currently teaches at the Nanyang Technological University. A chemical engineer by training, Dr. Lee feels that everyone benefits from a solid grounding in mathematics, and that this grounding can only be obtained through sufficient practice on a variety of question types.

The solutions to each question are developed with care with the idea of deepening the student's number sense and strengthening his basic technique of solving mathematical problems. To this end, great attention has been paid to the steps and explanations leading to the final answer.

Dr. Lee is the founder of several mathematics-related initiatives, and can be contacted at ascklee@gmail.com.

mathematics-dl.org