

# Solutions to Exercise 15.2

(Ho Soo Thong, David Khor Nyak Hiong and Yan  
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Solved by:

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**Exercise 15.2** (p. 381–382)

**Basic (Questions 1–2)**

**Question 1(a)**

$$y = 4x - 1$$

$$\frac{dy}{dx} = 4$$

**Question 1(b)**

$$y = x^2 + 2x$$

$$\begin{aligned}\frac{dy}{dx} &= 2x + 2 \\ &= 2(x + 1)\end{aligned}$$

**Question 1(c)**

$$y = x^{\frac{3}{2}} - 1$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{3}{2}x^{\frac{1}{2}} \\ &= \frac{3}{2}\sqrt{x}\end{aligned}$$

**Question 2(a)**

$$\begin{aligned}y &= -9 - x \\ \frac{dy}{dx} &= 0 - 1 \\ &= -1\end{aligned}$$

**Question 2(b)**

$$\begin{aligned}y &= -x^3 - 1 \\ \frac{dy}{dx} &= -3x^2 - 0 \\ &= -3x^2\end{aligned}$$

**Question 2(c)**

$$\begin{aligned}y &= x^6 - 5x \\ \frac{dy}{dx} &= 6x^5 - 5\end{aligned}$$

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### Intermediate (Questions 3–10)

#### Question 3(i)

$$y = x^3 + 3x$$

$$\frac{dy}{dx} = 3x^2 + 3$$

#### Question 3(ii)

It can be seen that  $\frac{dy}{dx}$  is always positive, whatever the value of  $x$ . This means that the function is always increasing.

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**Question 4**

$$y = \frac{2x}{x+1}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(x+1) \times \frac{d}{dx}(2x) - 2x \times \frac{d}{dx}(x+1)}{(x+1)^2} \\ &= \frac{(x+1) \times 2 - 2x \times 1}{(x+1)^2} \\ &= \frac{2x+2-2x}{(x+1)^2} \\ &= \frac{2}{(x+1)^2}\end{aligned}$$

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**Question 5**

$$\begin{aligned}y &= \frac{x}{x^2 + 1} \\ \frac{dy}{dx} &= \frac{(x^2 + 1) \times \frac{d}{dx}(x) - x \times \frac{d}{dx}(x^2 + 1)}{(x^2 + 1)^2} \\ &= \frac{(x^2 + 1) \times (1) - x \times (2x)}{(x^2 + 1)^2} \\ &= \frac{x^2 + 1 - 2x^2}{(x^2 + 1)^2} \\ &= \frac{1 - x^2}{(x^2 + 1)^2}\end{aligned}$$

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**Question 6(a)**

$$y = x^2 + 2x - 4$$

**Question 6(b)**

$$y = 2x^3 - 3x^2 + 6$$

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**Question 7(a)**

$$y = 3x^2 + 4x - 3$$

**Question 7(b)**

$$y = 2x^3 - 9x^2 + 12x - 3$$

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**Question 8(i)**

$$\begin{aligned}y &= (x + 1)^4 \\ \frac{dy}{dx} &= 4(x + 1)^3 \times \frac{d}{dx}(x + 1) \\ &= 4(x + 1)^3 \times 1 \\ &= 4(x + 1)^3\end{aligned}$$

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**Question 9(i)**

$$y = \frac{x^2 - 1}{x^2 + 1}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(x^2 + 1) \times \frac{d}{dx}(x^2 - 1) - (x^2 - 1) \times \frac{d}{dx}(x^2 + 1)}{(x^2 + 1)^2} \\ &= \frac{(x^2 + 1) \times (2x) - (x^2 - 1) \times (2x)}{(x^2 + 1)^2} \\ &= \frac{(2x^3 + 2x) - (2x^3 - 2x)}{(x^2 + 1)^2} \\ &= \frac{4x}{(x^2 + 1)^2}\end{aligned}$$

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**Question 10**

$$T = -5x^2 + 90x + 65$$

$$\frac{dT}{dx} = -10x + 90$$

$$-10x + 90 < 0$$

$$90 < 10x$$

$$9 < x$$

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### Advanced (Questions 11–13)

#### Question 11(a)

$$y = x^3$$
$$\frac{dy}{dx} = 3x^2$$

#### Question 11(b)

$$y = (x + 1)^3$$
$$\frac{dy}{dx} = 3(x + 1)^2 \times \frac{d}{dx}(x + 1)$$
$$= 3(x + 1)^2 \times (1)$$
$$= 3(x + 1)^2$$

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**Question 12(i)**


$$y = \frac{3x + 4}{\sqrt{2x - 1}}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{\sqrt{2x - 1} \times \frac{d}{dx}(3x + 4) - (3x + 4) \times \frac{d}{dx}(\sqrt{2x - 1})}{(\sqrt{2x - 1})^2} \\ &= \frac{\sqrt{2x - 1} \times 3 - (3x + 4) \times \left(\frac{1}{2}(2x - 1)^{-\frac{1}{2}} \times \frac{d}{dx}(2x - 1)\right)}{2x - 1} \\ &= \frac{3\sqrt{2x - 1} - (3x + 4) \left(\frac{1}{2}(2x - 1)^{-\frac{1}{2}} \times 2\right)}{2x - 1} \\ &= \frac{3\sqrt{2x - 1} - (3x + 4) \left(\frac{1}{\sqrt{2x - 1}}\right)}{2x - 1} \\ &= \frac{3\sqrt{2x - 1} - \frac{3x + 4}{\sqrt{2x - 1}}}{2x - 1} \\ &= \frac{\frac{3(2x - 1) - (3x + 4)}{\sqrt{2x - 1}}}{2x - 1} \\ &= \frac{6x - 3 - 3x - 4}{(2x - 1)\sqrt{2x - 1}} \\ &= \frac{3x - 7}{(2x - 1)^{\frac{3}{2}}} \end{aligned}$$

**Question 13(i)**

$$y = \frac{x^2 - 2ax - 2}{x - a}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(x - a) \times \frac{d}{dx}(x^2 - 2ax - 2) - (x^2 - 2ax - 2) \times \frac{d}{dx}(x - a)}{(x - a)^2} \\ &= \frac{(x - a) \times (2x - 2a) - (x^2 - 2ax - 2) \times (1)}{(x - a)^2} \\ &= \frac{2(x - a)(x - a) - (x^2 - 2ax - 2)}{(x - a)^2} \\ &= \frac{2(x - a)^2 - (x^2 - 2ax - 2)}{(x - a)^2} \\ &= \frac{2(x - a)^2}{(x - a)^2} - \frac{x^2 - 2ax - 2}{(x - a)^2} \\ &= 2 - \frac{x^2 - 2ax + a^2 - 2 - a^2}{(x - a)^2} \\ &= 2 - \frac{x^2 - 2ax + a^2 - (2 + a^2)}{(x - a)^2} \\ &= 2 - \frac{x^2 - 2ax + a^2}{(x - a)^2} - \frac{2 + a^2}{(x - a)^2} \\ &= 2 - \frac{(x - a)^2}{(x - a)^2} - \frac{2 + a^2}{(x - a)^2} \\ &= 2 - 1 - \frac{2 + a^2}{(x - a)^2} \\ &= 1 - \frac{2 + a^2}{(x - a)^2} \end{aligned}$$

 A portrait of Dr. Lee, a middle-aged man with short black hair, wearing glasses and a light-colored button-down shirt. He is looking directly at the camera with a neutral expression.	<p><a href="#">Dr. Lee</a> is an experienced teacher who has taught at the Singapore and Temasek Polytechnics. He currently teaches at the Nanyang Technological University. A chemical engineer by training, Dr. Lee feels that everyone benefits from a solid grounding in mathematics, and that this grounding can only be obtained through sufficient practice on a variety of question types.</p> <p>The solutions to each question are developed with care with the idea of deepening the student's number sense and strengthening his basic technique of solving mathematical problems. To this end, great attention has been paid to the steps and explanations leading to the final answer.</p> <p><a href="#">Dr. Lee</a> is the founder of several mathematics-related initiatives, and can be contacted at <a href="mailto:ascklee@gmail.com">ascklee@gmail.com</a>.</p>
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